MTH 111, Math for Architects, Final Exam, Spring 2014

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QUESTION 1. (i) Let $f(x) = -x^3 + 8x - 1$. The slope of the tangent line to the curve at the point $(1, 6)$

a. 5
b. 6
c. 13

(ii) Let $f(x) = -2x^3 + 24x + 1$. Then $f(x)$ increases on the interval

a. $x \in (-\infty, -2) \cup (2, \infty)$
b. $x \in (-\infty, -\sqrt{12}) \cup (\sqrt{12}, \infty)$
c. $x \in (-\sqrt{12}, \sqrt{12})$
d. $x \in (-2, 2)$

(iii) let $f(x) = 3e^{(x^2-x-2)} + 4x + 5643217689$. Then $f'(2)$

a. 13
b. 11
c. 9
d. 7

~~(iv) Let $f(x) = (x + 1)e^{(x-2)} + 3x + 14523$. Then~~

~~a. $f(x)$ has an inflection point at $x = -3$~~
~~b. $f(x)$ has an inflection point at $x = -2$~~
~~c. $f(x)$ has no inflection points~~
~~d. $f(x)$ has an inflection point at $x = -1$~~
~~e. $f(x)$ has an inflection point at $x = 2$~~

(v) Let $f(x) = -x(2x - 32)^7 + 1550$. Then

a. $f(x)$ has a local maximum at $x = 32/9$
b. $f(x)$ has a local maximum $x = 2$
c. $f(x)$ has a local minimum at $x = 32/9$
d. $f(x)$ has a critical value when $x = -16$

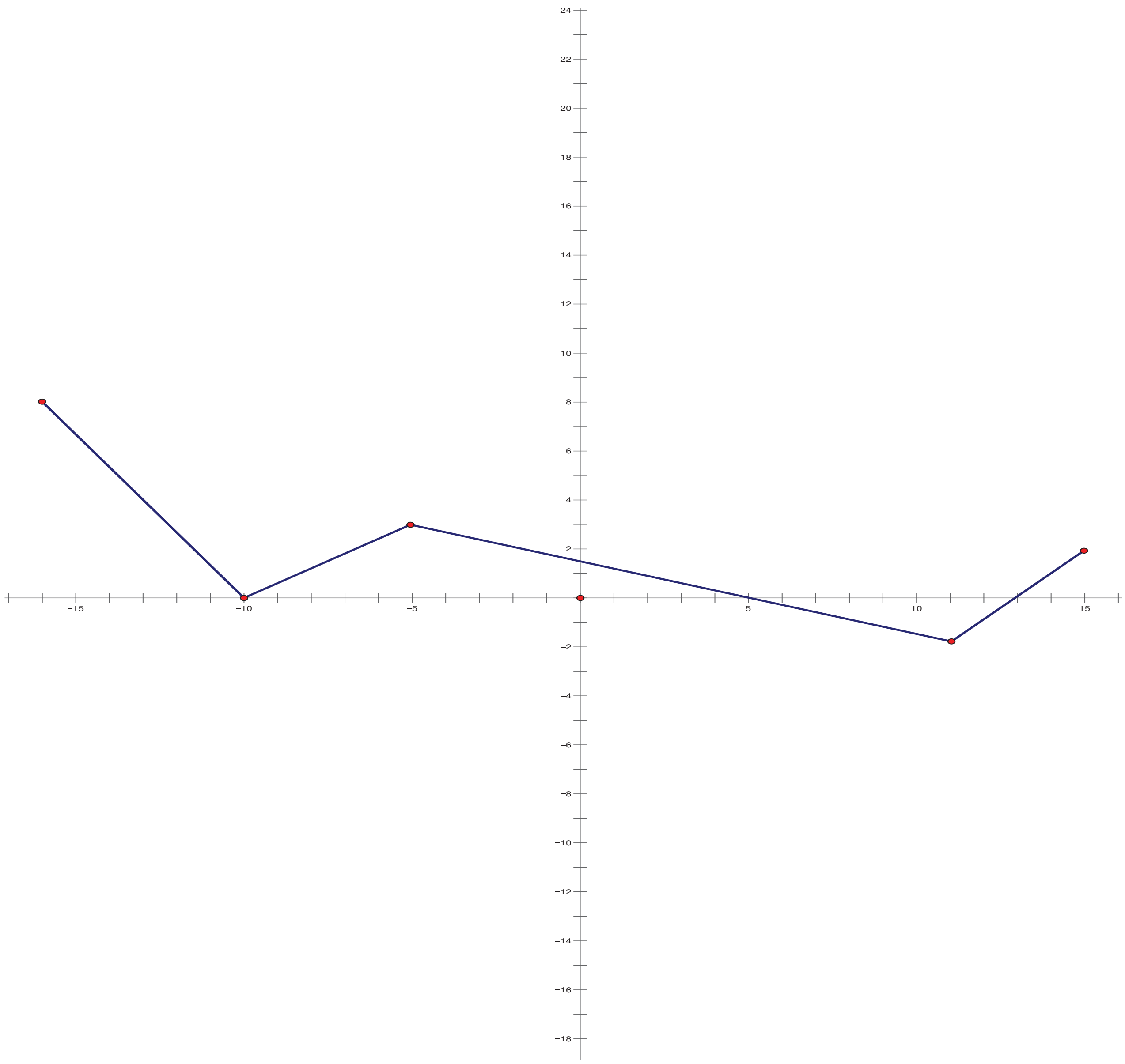
~~(vi) Given $x^2 + y^2 - xy + xe^y + ye^x - 203421897654 = 0$ Then $dy/dx =$~~

~~a. $\frac{2y-x+e^x}{y-2x-e^y}$~~
~~b. $\frac{y-2x-e^y}{x-2y-e^x}$~~
~~c. $\frac{2x-y+e^y}{2y-x+e^x}$~~
~~d. $\frac{y-2x-e^y}{2y-x+e^x}$~~

(vii) $lim_{x\to 3}\ \frac{x^2-10}{(x-2)^2} =$

a. -1
b. $-\infty$
c. ∞
d. DNE (does not exist)

(viii) Given the curve of $f'(x)$ on the interval [-16, 15] (i.e., $-16 \leq x \leq 15$). Then



a. $f(x)$ is decreasing on the the intervals $(-16, -10) \cup (-5, 11)$

b. $f(x)$ is decreasing on the interval $(5, 13)$

c. $f(x)$ is increasing on the intervals $(-10, -5) \cup (11, 15)$

d. $f(x)$ is increasing on the interval $(-16, 13)$

(ix) Using the curve of $f'(x)$ above. Then

a. $f(x)$ has a critical value at $x = -10$ but $f(x)$ has neither min. value nor max. value at $x = -10$.

b. $f(x)$ has a local max. value at $x = -5$ and a local min. value at $x = 11$ and $x = -10$.

c. $f(x)$ has a local min. value at $x = 5$

d. $f(x)$ has a local max. value at $x = 11$.

~~(x) Using the curve of $f'(x)$ above. Then~~

~~a. the curve of $f(x)$ must be concave down on the intervals $(-16, -10) \cup (-5, 11)$.~~

~~b. the curve of $f(x)$ must be concave up on only the interval $(-16, -5)$~~

~~c. the curve of $f(x)$ must be concave up on the intervals $(-16, -5) \cup (5, 15)$~~

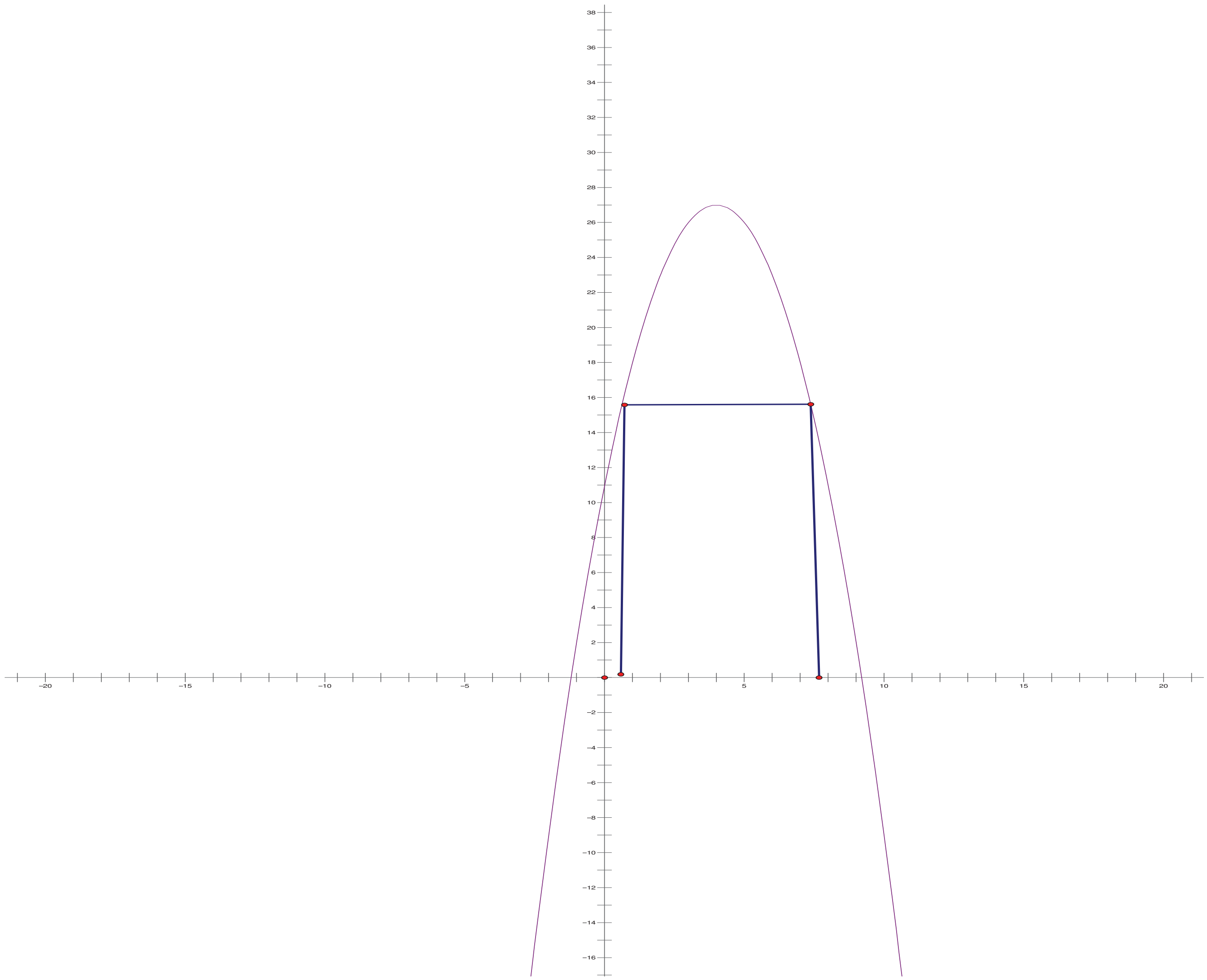
~~d. the curve of $f(x)$ must be concave down on the interval $(-10, 11)$~~

~~e. the curve of $f(x)$ must be concave down only on the interval $(-10, 5)$~~

(xi) Given x, y are two positive REAL numbers such that $xy = 3$ and $x + 12y$ is minimum. Then $x + 12y =$

a. 37
b. 15
c. 12
d. 6.5
e. 13

(xii) What is the area of the largest rectangle that can be drawn as in the figure below (note $f(x) = -x^2 + 8x + 11 = 27 - (x-4)^2$)?



a. 54
b. 144
c. 126
d. 216
e. 108

(xiii) the distance between the point $Q = (1, 1, 6)$ and the plane: $3x - 4z - 9 = 0$ is

a. 6
b. 2
c. 0.2
d. 1.2

(xiv) Given the points $A = (3, 4)$ and $B = (8, 10)$. What is the point c on the horizontal line $y = 2$ so that $|AC| + |CB|$ is minimum?

a. $(5\frac{1}{7}, 2)$
~~b.~~ (4, 2)
c. $(5\frac{6}{7}, 2)$
d. (3, 2)
e. (8, 2)
f. (5.5, 2)

(xv) Given $f(x) = ln[\frac{(4x-7)^5}{3x-5}]$. Then $f'(2)$

~~a.~~ 17
b. $\frac{20}{3}$
c. $\frac{4}{3}$
d. 4
e. 5
f. 2

(xvi) $lim_{x\to 2}\ \frac{e^{(4x-8)}+x^2-5}{x^3+x^2-12} =$

~~a.~~ 0.5
b. $\frac{5}{16}$
c. 0.25
d. 0.8
e. 0

(xvii) Let C be an arbitrary point on the ellipse $\frac{(x+1)^2}{4} + y^2 = 9$, and let c_1, c_2 be the foci of the ellipse. Then $|Cc_1| + |Cc_2| =$

~~a.~~ 4
b. 12
c. 6
d. 2

(xviii) Given the parabola $y = 0.1(x - 2)^2 - 2$. Then the directrix is

a. x = 4.5
b. y = 4.5
c. y = 2.5
~~d.~~ y = - 4.5
e. y = - 2.5

(xix) Let P be the parabola as in the above question. Given that the point $Q = (12, 8)$ lies on its curve, and C be its focus. Then $|QC|$ =

a. 7.5
b. 3.5
c. 5.5
~~d.~~ 12.5
e. 10.5

(xx) The intersection of the plane $x + y = 0$ and the plane $2x - z = 0$ is the following line.

~~a.~~ $x = -t, y = t, z = -2t$
b. $x = -t, y = -t, z = -2t$
c. $x = t, y = t, z = -2t$
d. $x = t, y = -t, z = -2t$

(xxi) Given the hyperbola $\frac{(x-1)^2}{9} - \frac{y^2}{16} = 1$. Let c_1, c_2 be the foci of the hyperbola, and C be an arbitrary point on the curve. Then $||Cc_1| - |Cc_2||$

a. 6
b. 10
c. 4
d. 8
e. 3
f. 5

(xxii) One of the following is a foci of the above hyperbola.

a. $(1, 5)$
b. $(-1, 5)$
c. $(6, 0)$
d. $(1 + \sqrt{7}, 0)$
e. $(1, \sqrt{7})$

(xxiii) $\int (2x - 7)^8\, dx =$

a. $\frac{(2x-7)^9}{18}$ + c
b. $\frac{(2x-7)^9}{4.5}$ + c
c. $\frac{(2x-7)^9}{9}$ + c
d. $\frac{(2x-7)^9}{2}$ + c

(xxiv) $\int\ xe^{(x^2+4)} + 4x - 1\ dx =$

a. $0.5e^{(x^2+4)} + 2x^2 - x + c$
b. $2e^{(x^2+4)} + x^2 - x + c$
c. $e^{(x^2+4)} + 2x^2 - x + c$
d. $2e^{(x^2+4)} + 2x^2 - x + c$

(xxv) Given $f'(x) = 4e^{(2x-6)} + 4x + 1$ and $f(3) = 27$. Then $f(x) =$

a. $4e^{(2x-6)} + 2x^2 + x + 2$
b. $2e^{(2x-6)} + 2x^2 + x + 4$
c. $e^{(2x-6)} + 2x^2 + x + 5$
d. $0.5e^{(2x-6)} + 2x^2 + x + 5.5$

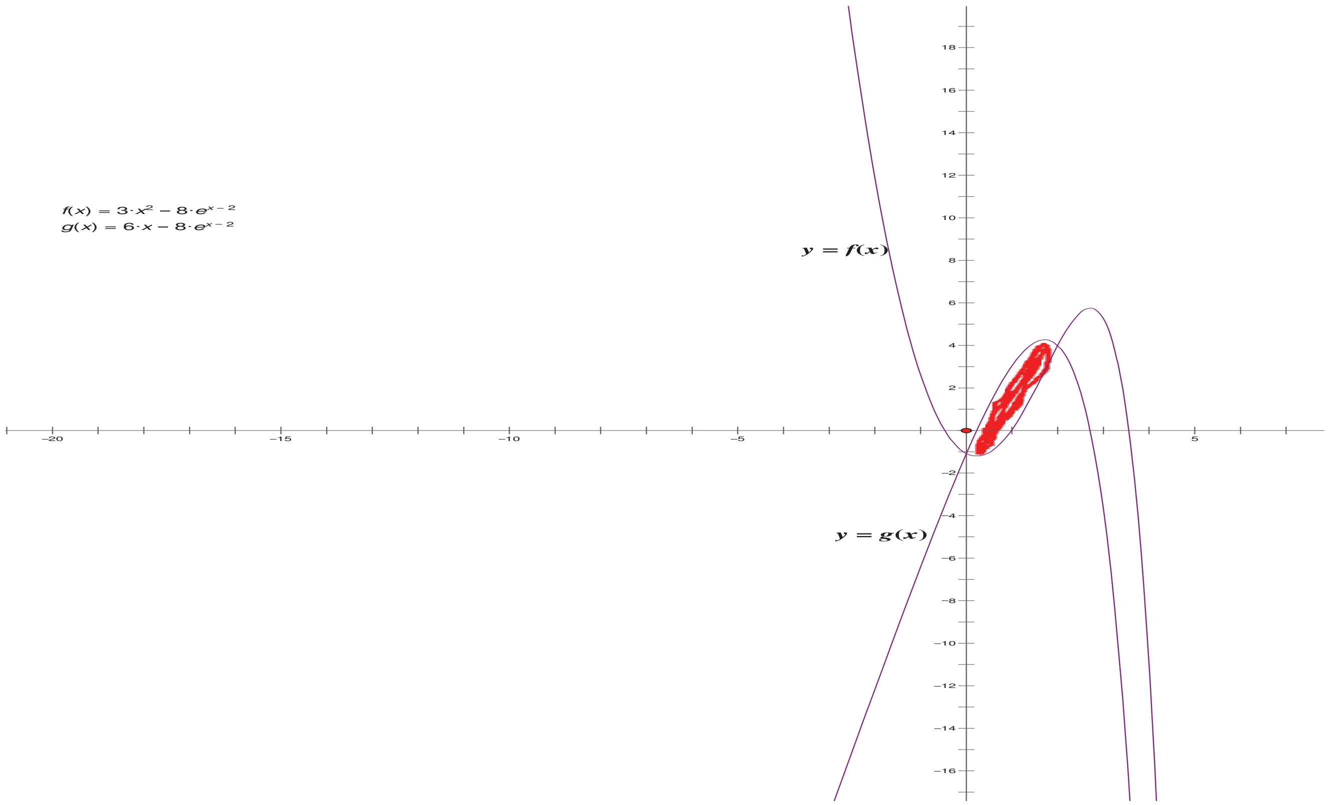
(xxvi) one of the following vectors can be drawn inside the plane : $3x - 2y + z = 4$

a. $2i + 3j + k$
b. $3i - 2j + k$
c. $3i - 2j - k$
d. $3i - 2j - 13k$

(xxvii) Given the points $(0, 1, 1), (0,\ 2.5\, 3), (2, 1, 1)$ are vertices of a triangle. The area of the triangle is :

a. 25
b. 12.5
c. 5
d. 2.5

(xxviii) Find the area of the region bounded by $f(x) = 3x^2 - 8e^{(x-2)}$ and $g(x) = 6x - 8e^{(x-2)}$ (see the region below and notice that the region is between $0 \leq x \leq 2$)



a. 20
b. 4
c. 36
d. 16

(xxix) One of the following lines lies entirely inside the plane: $x + 2y + 3z - 6 = 0$

a. $x = 1 - 2t, y = 1 + 4t, z = 1 - 2t$
b. $x = 1 + 3t, y = 1 + 2t, z = 1$
c. $x = 1 + t, y = 1 + 2t, z = 1 + 3t$
d. $x = t, y = t, z = 3 - t$

(xxx) $L_1 : x = t, y = 2t, z = 1 + t$, $L_2 : x = 1 + 2s, y = 2 - s, z = 3 + 2s$. Then

a. L_1 intersects L_2 at $(1, 2, 3)$
b. L_1 intersects L_2 at $(1, 1, 2)$
c. L_1 intersects L_2 at $(1, 2, 2)$
d. L_1 does not intersect L_2

(xxxi) Given $(0, 1, 1), (0, 2, 3), (2, 1, 1)$ lie in a plane P. Then an equation of P is

a. $-4(y - 1) - 2(z - 1) = 0$
b. $2x + 4(y - 1) - 2(z - 1) = 0$
c. $4(y - 1) - 2(z - 1) = 0$
d. $4(y - 1) + 2(z - 1) = 0$
e. $-2x - 4(y - 1) - 2(z - 1) = 0$

Faculty information

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